Chapter 9
Bayesian Econometrics: Detailed Outline

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Abstract This chapter will outline the evolution of Bayesian methods in econometrics. Since moving from the fields of probability and statistics, the Bayesian approach has made contributions in most areas of econometrics. While improved computational power has played an important role in Bayesian econometrics, this chapter will highlight advances due to improved computational algorithms as well as developments in model specification and applications that have both advanced the adoption of Bayesian methods. The chapter will identify the techniques have endured and found wide application, as well as those that have been replaced by newer methods and that that have fallen into disuse and found new purpose. There will be some discussion of contributions to empirical inference on questions important in economics.

9.1 Introduction

Brief overview of the Bayesian approach and then of the sections with in the chapter. The first subsection will discuss early advocates and adopters that brought the Bayesian approach into econometrics. Next, we will outline the early computational advances and important algorithms, some of which have passed into relative disuse, the reasons for which we will cover. Some of these techniques have found new purpose as models and data have evolved. To help in understanding the significance of these techniques, we devote a number of sections discussing their adoption into various areas in econometrics such as macroeconometrics/time series, microeconometrics covering applications in panel data, finance, ... etc. We will discuss important contributions (applications and influential new techniques and algorithms) to the scope of opportunities for inference and to our understanding of economics. The chapter will conclude with a discussion of potential new directions

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and areas of application.

Reference and reviews:

- Koop, Poirier and Tobias (2007): A comprehensive textbook that has shaped teaching and practice in the field.
- Geweke, Koop and Van Dijk (2011): Contains chapters by leading scholars on Bayesian methods in macro, micro, finance, and marketing.
- Basturk, Cakmakli, Ceyhan and van Dijk (2014) Analyzes publication and citation patterns across major journals.

9.2 The early adoption of Bayesian methods into econometrics

Brief overview of the early work adopting the Bayesian methodology into econometrics, moving from probability theory through statistics generally and into econometrics specifically. This section will point to key actors (such as Drèze, Zellner, Maddala, Lindley, Kloek and van Dijk) in the adoption of a Bayesian approach and the technical challenges they overcame but also those that prevented a more general uptake in econometric analysis.

This section will also give a simple introduction to the Bayesian approach in a general form, including building a posterior, approaches to model comparison and theoretical expressions for important objects of interest including posterior probabilities, the Bayes factor and marginal likelihoods.

Attention will be given to early approaches to inference including the purely analytical methods and numerical approximations to integrals. Discuss how the employment of these approaches have changed over time, some are little used now while others are used in new ways.

The origins:

- Bayes (1763) introduced Bayes' Theorem, laying the groundwork for probabilistic inference.
- Pierre-Simon Laplace greatly expanded upon Bayes' ideas, applying them to astronomy and statistics.

From probability framework to econometric models: the middle of the 20th Century:

- Building upon Lapace's method, Leonard J. Savage, Bruno de Finetti, and Dennis Lindley helped formalize Bayesian decision theory.

- Arnold Zellner played a pivotal role in applying Bayesian methods to econometrics and did much to promote its adoption, especially in regression and simultaneous equations models. Zellner developed the Seemingly Unrelated Regressions (SUR) model and contributed to Bayesian model comparison and prediction. His 1971 textbook is still a useful first reference for graduate students moving into Bayesian econometrics.
  - Zellner (1971) – "An Introduction to Bayesian Inference in Econometrics" A seminal book that laid the groundwork for Bayesian econometrics.

Examples of early adopters:

- Thomas Sargent - Nobel laureate applied Bayesian methods in multivariate macroeconomics and known for his work in rational expectations. Outline how Bayesian methods were employed to help his investigations.

- Christopher Sims - Nobel laureate who introduced the vector autoregression model and the attendant methodology for structural analysis (from a reduced form model). He was a strong advocate for Bayesian approaches in macroeconomic modeling. Famous quote: "Econometrics should always and everywhere be Bayesian."

- Herman K. van Dijk - With an early application of simulation for inference, he helped introduce such methods to econometrics. Link importance sampling to other methods being used in statistics.

Many researchers adopt Bayesian methods to complement frequentist methods, or to overcome, or avoid, challenges with frequentist methods. Examples of leading researchers using a mix of frequentist and Bayesian methods:

- James Heckman - Nobel laureate who used Bayesian methods in labor economics and microeconometrics.

- George Tauchen - Known for Bayesian inference in financial econometrics and time series analysis.

9.3 Early computational advances

The objective of Bayesian inference is the posterior distribution and expectations under this distribution. It is, therefore, necessary to have means to explore the posterior. Outline how the spread of greater computational power from mainframes to desktops permitted access to simulation algorithms that permitted inference using Bayesian methods be applied to a wider range of problems and models. Much of the discussion in this section, however, will be around the algorithmic advances and the authors that brought these to the attention of econometricians and demonstrated their power in exploring the posterior. Include examples on the use of data augmentation and latent data/variables, parameter expansion, unobserved components, mixtures (finite and infinite), hierarchical models, etc.. Advances in the algorithmic techniques continue today and approaches have proliferated, but later sections will incorporate discussion of areas of application and include discussion of model selection such as estimation of Bayes factors, marginal likelihoods, predictive densities, information criteria etc.

The objective of Bayesian inference is the posterior distribution, $p(\theta|y)$, and expectations under this distribution, $g_y = \int g(\theta)p(\theta|y)(d\theta)$. The posterior is constructed from the likelihood, $L(y|\theta)$, and the prior distribution for θ, $p(\theta)$, as

$$p(\theta|y) = \frac{L(y|\theta)p(\theta)}{m_y}$$

where

$$m_y = \int L(y|\theta)p(\theta)(d\theta)$$

is known as the marginal likelihood. Another important object of interest is the marginal posterior for a subset of the parameters. For example, if θ

This chapter will focus upon early methods to estimate g_y and $p(\beta_0|y)$ in a range of applications. Techniques covered will include the Importance sample, the Metropolis-Hastings algorithm including the Gibbs sampler, the utility of latent structures such as data augmentation and hierarchical models.

Important references:

- Drèze (1974)
- Kloek and van Dijk (1978)
- Schwarz (1978)
- Geweke (1989)
- Gelfand, Alan E., and Adrian F. M. Smith. “Sampling-Based Approaches to Calculating Marginal Densities.” Journal of the American Statistical Association

85, no. 410 (1990): 398–409.

- Steel, Mark F. J. & Richard, Jean-Francois, 1991. ”Bayesian multivariate exogeneity analysis : An application to a UK money demand equation,” Journal of Econometrics, Elsevier, vol. 49(1-2), pages 239-274.

- Julian Besag & Jeremy York & Annie Mollié, 1991. ”Bayesian image restoration, with two applications in spatial statistics,” Annals of the Institute of Statistical Mathematics, Springer;The Institute of Statistical Mathematics, vol. 43(1), pages 1-20, March.

- Chib, Siddhartha and Greenberg, Edward (1995). ”Understanding the Metropolis–Hastings Algorithm”. American Statistician, 49(4), 327–335.

- Sims, Christopher A & Zha, Tao, 1998. ”Bayesian Methods for Dynamic Multivariate Models,” International Economic Review, Department of Economics, University of Pennsylvania and Osaka University Institute of Social and Economic Research Association, vol. 39(4), pages 949-968, November.

- Kadiyala, K Rao & Karlsson, Sune, 1997. ”Numerical Methods for Estimation and Inference in Bayesian VAR-Models,” Journal of Applied Econometrics, John Wiley & Sons, Ltd., vol. 12(2), pages 99-132, March-Apr.

- John Geweke, 1999. ”Using simulation methods for Bayesian econometric models: inference, development, and communication,” Econometric Reviews, Taylor & Francis Journals, vol. 18(1), pages 1-73.

- Fruhwirth-Schnatter, S. (2006). Finite Mixture and Markov Switching Models (1st ed. 2006.). Springer New York.

- Fruhwirth-Schnatter, S., Celeux, G., & Robert, C.P. (Eds.). (2019). Handbook of Mixture Analysis (1st ed.). Chapman and Hall/CRC.

To demonstrate the power of some of the above techniques, the section will draw upon a nice example of using a Bayesian approach that resulted in inference not possible from a frequentist approach. The objects of interest, the expectations g, are highly non-linear:

- Geweke, John. “The Secular and Cyclical Behavior of Real GDP in 19 OECD Countries, 1957-1983.” Journal of Business & Economic Statistics 6, no. 4 (1988): 479–86. https://doi.org/10.2307/1391467.

9.4 Model comparison and model and variable selection

Model comparison and model selection are fundamental aims in econometrics. These concepts capture the simplest questions such as evidence on a linear restriction, to comparing alternative economic theories. Often, interest is not in the restriction or even the model, but rather some outcome (such as a forecast) or other object (such as an elasticity). In this case, it is often preferable to avoid conditioning on any one model to improve the external validity of the inference and this naturally leads to the concept of Bayesian model averaging.

Recall the notation in Section 9.3. Suppressed in the notation in Section 9.3 is the model used to form the likelihood. We will denote a generic first model by M_1. Such a model may be a simple as a linear regression model

$$y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \varepsilon \text{ where } \varepsilon \sim iidN(0,\sigma^2)$$

To compare the empirical support for the model subject to $\beta_2 = 0$, we would label this another model, M_0 say, and compute the posterior probabilities for the two models. First we need to augment the notation to account for the models. For model $i \in \{0,1\}$, let the likelihood be $L(\theta_i|M_i, y)$ where $\theta_0 = (\beta_0, \beta_1, \sigma)$ and $\theta_1 = (\beta_0, \beta_1, \beta_2, \sigma)$, the prior $p(\theta_i|M_i)$ and specify a prior probability for M_i, $0 \leq P(M_i) \leq 1$.

$$p(\theta|M_i, y) = \frac{L(y|\theta, M_i)p(\theta|M_i)}{m_i}$$

where now

$$m_i = \int L(y|\theta, M_i)p(\theta|M_i)(d\theta)$$

Note the special treatment and different algorithms that exist when there is an *encompassing model* such as in this case where M_1 encompasses M_0. That is, we obtain M_0 by imposing the restriction $\beta_2 = 0$ in M_1.

This section will outline a number of important techniques, and their limitations, developed to estimate m_i.

Main points to make in discussing approaches to model averaging, model selection and model comparison:

- The objects of interest in model comparison/selection.
- Cases where a closed form analytical expression exists.
- The harmonic mean estimator, problems and the range of alternatives developed to address the limitations.

- The Savage-Dickey density ratio for estimating the Bayes factor - when applicable and examples of applications.
- The Chib method of estimating the marginal likelihood.
- Bayesian model averaging.
- Reversible jump.
- Predictive densities.
- ...

Foundational and Methodological Papers

- Kass, Robert E.; Tierney, Luke; Kadane, Joseph B. (1991). ”Laplace’s method in Bayesian analysis”. Statistical Multiple Integration. Contemporary Mathematics. Vol. 115. pp. 89–100.
- George, E. I., & McCulloch, R. E. (1993). Variable Selection via Gibbs Sampling. Journal of the American Statistical Association, 88(423), 881–889.
- Fernandez, Carmen & Ley, Eduardo & Steel, Mark F. J., 2001. “Benchmark priors for Bayesian model averaging,” Journal of Econometrics, Elsevier, vol. 100(2), pages 381-427, February
- Spiegelhalter, David J.; Best, Nicola G.; Carlin, Bradley P.; van der Linde, Angelika (2002). ”Bayesian measures of model complexity and fit (with discussion)”. Journal of the Royal Statistical Society, Series B. 64 (4): 583–639.
- Spiegelhalter, David J.; Best, Nicola G.; Carlin, Bradley P.; van der Linde, Angelika (2014). ”The deviance information criterion: 12 years on (with discussion)”. Journal of the Royal Statistical Society, Series B. 76 (3): 485–493.
- Håvard Rue & Sara Martino & Nicolas Chopin, 2009. “Approximate Bayesian inference for latent Gaussian models by using integrated nested Laplace approximations,” Journal of the Royal Statistical Society Series B, Royal Statistical Society, vol. 71(2), pages 319-392, April.
- Verdinelli, Isabella, and Larry Wasserman. “Computing Bayes Factors Using a Generalization of the Savage-Dickey Density Ratio.” Journal of the American Statistical Association 90, no. 430 (1995): 614–18.

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- Dickey, J., and Lientz, B. P. (1970), “The Weighted Likelihood Ratio, Sharp Hypotheses About Chances, the Order of a Markov Chain,” Annals of Mathematical Statistics, 41, 214-226.
- Guñel, E., and Dickey, J. (1974), “Bayes Factors for Independence in Contingency Tables,” Biometrika, 61, 545-557.

- Victor Chernozhukov – “Bayesian Econometrics” (MIT Lecture Notes) Offers a concise and rigorous overview of Bayesian principles and their econometric applications.
- David J. Spiegelhalter & Nicola G. Best & Bradley P. Carlin & Angelika Van Der Linde, 2002. “Bayesian measures of model complexity and fit,” Journal of the Royal Statistical Society Series B, Royal Statistical Society, vol. 64(4), pages 583-639, October.
- Xavier Sala-I-Martin & Gernot Doppelhofer & Ronald I. Miller, 2004. “Determinants of Long-Term Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach,” American Economic Review, American Economic Association, vol. 94(4), pages 813-835, September
- John Geweke, 1999. “Using simulation methods for bayesian econometric models: inference, development,and communication,” Econometric Reviews, Taylor & Francis Journals, vol. 18(1), pages 1-73.
- Philip J. Brown. Jim E. Griffin. “Inference with normal-gamma prior distributions in regression problems.” Bayesian Anal. 5 (1) 171 - 188, March 2010.
- Raffaella Giacomini, Toru Kitagawa, Matthew Read (2021) – “Robust Bayesian Analysis for Econometrics” Advances Bayesian robustness and sensitivity analysis in structural models.
- Gelfand, A. E., and D. K. Dey. “Bayesian Model Choice: Asymptotics and Exact Calculations.” Journal of the Royal Statistical Society. Series B (Methodological) 56, no. 3 (1994): 501–14.
- Chib, Siddhartha (1995). ”Marginal Likelihood from the Gibbs Output”. Journal of the American Statistical Association, 90(4), 1313–1321.

9.5 Time series applications

This section will follow the general evolution of macroeconometrics from structural models (Cowles Foundation) to reduced form models (the VAR and Structural VAR) and back to structure from micro foundations (the DSGE with the VAR(MA) approximations to DSGE models). Include problems and areas of application unique to finane. In both the discussion of macroeconomics and finance, the focus will be on the contributions from the Bayesian approach. This be coupled with identifying the important contributions to our understanding of economic and financial problems that have come from these papers.

Examples of influential applications in VAR and Structural VAR, TVP and structural instability:

- Sims, Christopher A. “Macroeconomics and Reality.” Econometrica 48, no. 1 (1980): 1–48.
- Sims, C. A. (1980b), Comparison of Interwar and Postwar Business Cycles: Monetarism Reconsidered, American Economic Review 70 (2), 250-257.
- Doan, T., Litterman, R., & Sims, C. (1984). Forecasting and conditional projection using realistic prior distributions. Econometric Reviews, 3(1), 1–100.
- Litterman, Robert B, 1986. ”Forecasting with Bayesian Vector Autoregressions-Five Years of Experience,” Journal of Business & Economic Statistics, American Statistical Association, vol. 4(1), pages 25-38, January.
- Litterman, Robert, 1986. ”Forecasting with Bayesian vector autoregressions – Five years of experience : Robert B. Litterman, Journal of Business and Economic Statistics 4 (1986) 25-38,” International Journal of Forecasting, Elsevier, vol. 2(4), pages 497-498.
- Timothy Cogley, Thomas J. Sargent, Drifts and volatilities: monetary policies and outcomes in the post WWII US, Review of Economic Dynamics, Volume 8, Issue 2, 2005, Pages 262-302.
- Timothy Cogley and Thomas J. Sargent, 2001 Evolving Post-World War II U.S. Inflation Dynamics, NBER Macroeconomics Annual 16:, 331-373
- Giorgio E. Primiceri, 2005 Time Varying Structural Vector Autoregressions and Monetary Policy, The Review of Economic Studies, Volume 72, Issue 3, July 2005, Pages 821–852.

- Chang-Jin Kim & Charles R. Nelson, 1999. ”Has The U.S. Economy Become More Stable? A Bayesian Approach Based On A Markov-Switching Model Of The Business Cycle,” The Review of Economics and Statistics, MIT Press, vol. 81(4), pages 608-616, November.

- Sims, Christopher, A., and Tao Zha. 2006. ”Were There Regime Switches in U.S. Monetary Policy?” American Economic Review 96 (1): 54–81..

- Marta Banbura & Domenico Giannone & Lucrezia Reichlin, 2010. ”Large Bayesian vector auto regressions,” Journal of Applied Econometrics, John Wiley & Sons, Ltd., vol. 25(1), pages 71-92.

- Koop, Gary & Korobilis, Dimitris, 2010. ”Bayesian Multivariate Time Series Methods for Empirical Macroeconomics,” Foundations and Trends(R) in Econometrics, now publishers, vol. 3(4), pages 267-358, July.

Stochastic volatility:

- Jacquier, Eric, Nicholas G. Polson, and Peter E. Rossi. “Bayesian Analysis of Stochastic Volatility Models.” Journal of Business & Economic Statistics 12, no. 4 (1994): 371–89.

- Jacquier, Eric & Polson, Nicholas G & Rossi, Peter E, 2002. ”Bayesian Analysis of Stochastic Volatility Models,” Journal of Business & Economic Statistics, American Statistical Association, vol. 20(1), pages 69-87, January.

- Jacquier, Eric & Polson, Nicholas G & Rossi, Peter E, 1994. ”Bayesian Analysis of Stochastic Volatility Models: Comments: Reply,” Journal of Business & Economic Statistics, American Statistical Association, vol. 12(4), pages 413-417, October.

- Sangjoon Kim & Neil Shephard & Siddhartha Chib, 1998. ”Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models,” The Review of Economic Studies, Review of Economic Studies Ltd, vol. 65(3), pages 361-393.

Examples of applications in DSGE:

- Marco Del Negro & Frank Schorfheide 2004 “Priors from General Equilibrium Models for VARs” A key paper in Bayesian macroeconometrics using DSGE-informed priors.

- Frank Smets & Rafael Wouters, 2007. ”Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” American Economic Review, American

Economic Association, vol. 97(3), pages 586-606, June.

- Sungbae An & Frank Schorfheide, 2007. ”Bayesian Analysis of DSGE Models,” Econometric Reviews, Taylor & Francis Journals, vol. 26(2-4), pages 113-172.

- Sungbae An & Frank Schorfheide, 2007. ”Bayesian Analysis of DSGE Models—Rejoinder,” Econometric Reviews, Taylor & Francis Journals, vol. 26(2-4), pages 211-219.

- Adolfson, Malin & Laseen, Stefan & Linde, Jesper & Villani, Mattias, 2007. ”Bayesian estimation of an open economy DSGE model with incomplete pass-through,” Journal of International Economics, Elsevier, vol. 72(2), pages 481-511, July.

Finance:

- Michael Johannes and Nicholas Polson, 2010, CHAPTER 13 - MCMC Methods for Continuous-Time Financial Econometrics, Editor(s): YACINE AÏT-SAHALIA, LARS PETER HANSEN, In Handbooks in Finance, Handbook of Financial Econometrics: Applications, Elsevier, Volume 2, Pages 1-72.

- Joshua C.C. Chan, Caterina Santi, Speculative bubbles in present-value models: A Bayesian Markov-switching state space approach, Journal of Economic Dynamics and Control, Volume 127, 2021.

9.6 Applications in microeconomics

This section will follow the general evolution of microeconometrics and the contributions from Bayesian work on a variety of processes, such as limited dependent variables (including binary, multinomial), duration models, hierarchical models, panel data, hierarchical linear models, treatment of endogeneity and weak instruments, choice modelling, count and binary data, ordinal data, treatment effects, hurdle/sample selection models, Dirichlet processes. Areas of application discussed include returns to education, labour more generally, health care, health insurance, health outcomes.

Useful reference material

- Li, Mingliang & Tobias, Justin L. (2010/2011). Bayesian Methods in Microeconometrics. Reviews Bayesian analysis of microeconometric models (linear regression, probit/logit/Tobit, hierarchical models, treatment effects, endogeneity, count and duration models) with discussion of MCMC methods.

- Li, Minglian & Tobias, Justin (2012). Bayesian Methods in Microeconometrics, in The Oxford Handbook of Bayesian Econometrics, Ch. 6.
  An overview of Bayesian computational approaches to major microeconometric models, including hierarchical and nonlinear latent-variable models, multinomial/multivariate probit, and Bayesian treatment-effects models.

- Chan, Joshua & Tobias, Justin (2020). Bayesian Econometric Methods. Although broader than microeconometrics, this is a widely used reference chapter illustrating Bayesian inference in micro-applications (binary choice models, treatment-response models, nonparametric Bayes via Dirichlet process).

- Geweke, J., Koop, G., & Van Dijk, H. (Eds.) (2011). The Oxford Handbook of Bayesian Econometrics. Contains multiple chapters relevant to microeconometrics, including Bayesian modeling, MCMC simulation, and flexible/nonparametric methods.

- van Dijk, Herman (2023). Challenges and Opportunities for 21st Century Bayesian Econometricians. While general, this discussion paper outlines modern Bayesian simulation strategies and modeling complexity that underpin contemporary microeconometric applications.

Example important references

- Zellner , A. and Rossi , P. E. 1984 . Bayesian Analysis of Dichotomous Quantal Response Models . Journal of Econometrics , 25 : 365 – 393.

- Albert, J. H., & Chib, S. (1993). Bayesian Analysis of Binary and Polychotomous Response Data. Journal of the American Statistical Association, 88(422), 669–679.

- Chib , S. 1992 . Bayes Inference in the Tobit Censored Regression Model . Journal of Econometrics , 51 : 79 – 99.

- P. Dellaportas, A. F. M. Smith, Bayesian Inference for Generalized Linear and Proportional Hazards Models Via Gibbs Sampling, Journal of the Royal Statistical Society Series C: Applied Statistics, Volume 42, Issue 3, September 1993, Pages 443–459.

- Imbens, Guido W., and Donald B. Rubin. “Bayesian Inference for Causal Effects in Randomized Experiments with Noncompliance.” The Annals of Statistics 25, no. 1 (1997): 305–27.

- Martin Burda, Matthew Harding, Jerry Hausman, A Bayesian mixed logit–probit model for multinomial choice, Journal of Econometrics, Volume 147, Issue 2, 2008, Pages 232-246.

9.7 Applications in marketing

This section outline the range of models in which Bayesian methods have assisted with, or enabled, inference. These models will over lap with many of those discussed in the section on microeconometrics, so we will emphasize models such as the Bayesian Market Mix Modelling, decision theory, with will follow the general evolution of microeconometrics from work with a variety of processes, such as limited dependent variables (including binary, multinomial), duration models, hierarchical models, panel data, hierarchical linear models, treatment of endogeneity and weak instruments, choice modelling, count and binary data, ordinal data, treatment effects, hurdle/sample selection models, Dirichlet processes. Areas of application discussed include returns to education, labour more generally, health care, health insurance, health outcomes.

Examples of applications in marketing:

- Campolieti, Michele. “Bayesian Estimation and Smoothing of the Baseline Hazard in Discrete Time Duration Models.” The Review of Economics and Statistics 82, no. 4 (2000): 685–94.
- Green, Paul E.; Frank, Ronald E. (1966). ”Bayesian Statistics and Marketing Research”. Journal of the Royal Statistical Society. Series C (Applied Statistics). 15 (3): 173–190.
- Chernoff, H. and Moses, L. E. (1959). Elementary Decision Theory. New York: Wiley; London: Chapman & Hall.
- Schlaifer, R. (1959). Probability and Statistics for Business Decisions, New York: McGraw Hill.
- Rossi PE, Allenby GM, McCulloch R (2005). Bayesian Statistics and Marketing. Wiley.
- Roberts, Harry V. (1960). ”The New Business Statistics”. The Journal of Business. 33 (1): 21–30.
- Pratt, John W.; Raiffa, Howard; Schlaifer, Robert (June 1964). ”The Foundations of Decision under Uncertainty: An Elementary Exposition”. Journal of the

American Statistical Association. 59 (306): 353–375.

- Rossi, Peter E.; Allenby, Greg M. (August 2003). ”Bayesian Statistics and Marketing”. Marketing Science. 22 (3): 304–328.

- Rossi, P. E., Allenby, G. M., & Misra, S. (2024). Bayesian Statistics and Marketing (2nd ed.). John Wiley & Sons, Incorporated.

9.8 Conclusion

Give an impression of the current state of play – the many areas of development happening in Bayesian econometrics. Possible future directions.

References

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Bayes, T. (1763, 12). Lii. an essay towards solving a problem in the doctrine of chances. by the late rev. mr. bayes, f. r. s. communicated by mr. price, in a letter to john canton, a. m. f. r. s. *Philosophical Transactions*(53), 370-418. Retrieved from https://doi.org/10.1098/rstl.1763.0053 doi: 10.1098/rstl.1763.0053

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Kloek, T. & van Dijk, H. K. (1978, January). Bayesian estimates of equation system parameters: An application of integration by monte carlo. *Econometrica*, *46*(1), 1-19.

Koop, G., Poirier, D. J. & Tobias, J. L. (2007). *Bayesian econometric methods*. Cambridge University Press.

Schwarz, G. E. (1978). Estimating the dimension of a model. *Annals of Statistics*, *6*, 461-464.

Zellner, A. (1971). *An introduction to bayesian inference in econometrics*. New York: J. Wiley.